CALCULATION OF THE CHARACTERISITICS OF ELECTRIC-ARC HEATING OF A GAS

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There have been several theoretical investigations [1, 2] in which the electrical and enthalpy characteristics of a cylindrical arc have been obtained. In the investigation of electric-arc heating, the gas-dynamic characteristics are required as well.

Below we investigate the electric-arc heating of gas in a round tube for the case in which the local enthalpy and specific flow rate of the gas are independent of the longitudinal coordinate (stabilized electric-arc heating). We give relationships for the numerical determination of the electrical and enthalpy characteristics by the method of successive approximations, and also relationships for the subsequent determination of the gasdynamic characteristics. The results are given of calculations for air at pressure of 1-100 atmospheres (1 atm = $1.01325 \cdot 10^5$ N/m^2) and a temperature of 6000° K on the column axis.

1. We consider a laminar flow of gas in a cylindrical tube of round cross section in the presence of the positive column of an electric arc for the case in which the local enthalpy h and the specific flow rate ρv_z are independent of the longitudinal coordinate z. (This case was mentioned in [3], but there was a misprint – ρ, v_z instead of ρv_z). Radiation is ignored. Then the functions

$$\Psi = \int_0^r \rho v_z \, r dr, \qquad \Phi = \int_0^r \rho v_z h r dr$$

are independent of z. The pressure along the tube falls and, hence, the temperature T and the function

$$s = \int_0^T \lambda dT$$

(λ is the thermal conductivity) will vary slightly with z, but in practice they can be regarded as constant. Owing to the possible variation of v_z along the tube, the function

$$\Lambda = \int_0^r \rho v_z{}^2 \, r dr$$

will also depend on z.

For this case the approximate momentum and energy equations given in [3] take the form

$$\mu r \frac{\partial v_z}{\partial r} = \frac{r^2}{2} \frac{dp}{dz} + \frac{\partial \Lambda}{\partial z}, \qquad r \frac{\partial s}{\partial r} = -E^2 \int_0^r \sigma r \, dr. \qquad (1.1)$$

The boundary conditions are

$$v_z = 0, \qquad h = h_w = \text{const} \qquad \text{when} \quad r = r_w. \qquad (1.2)$$

The subscript w indicates the parameters on the wall.

In view of thermochemical equilibrium we regard the quantities ρ, h, λ, μ, and σ as known functions of temperature and pressure.

In conjunction with Eqs. (1.1) we must consider the expressions for the total current in the tube and the gas flow rate, respectively,

$$I = 2\pi E \int_0^{r_d} \sigma \, r dr, \qquad G = 2\pi \int_0^{r_w} \rho v_z \, r dr. \qquad (1.3)$$

The subscript d indicates the boundary of the conducting region. For the conducting region ($0 \le r \le r_d$) the energy equation (1.1) is represented in integral form:

$$s^\circ = s_d{}^\circ + \frac{\sigma_0 (E r_w)^2}{S_0} [F_2(\eta_d) - F_2(\eta)], \quad s^\circ = \frac{s}{s_0}, \quad \sigma^\circ = \frac{\sigma}{\sigma_0}$$

$$F_2(\eta) = \int_0^\eta \frac{F_1(\eta) \, d\eta}{\eta}, \qquad F_1(\eta) = \int_0^\eta \sigma_0 \, \eta d\eta, \qquad \eta = \frac{r}{r_w}. \quad (1.4)$$

The subscript 0 corresponds to the value $\eta = 0$.

When $\eta = 0$ it follows from Eq. (1.4) that

$$E r_w = \left(\frac{s_0 - s_d}{\sigma_0 F_2(\eta_d)} \right)^{1/2}. \qquad (1.5)$$

From Eq. (1.3), using (1.5), we obtain

$$\frac{I}{r_w} = 2\pi F_1(\eta_d) \left(\frac{\sigma_0 (s_0 - s_d)}{F_2(\eta_d)} \right)^{1/2}. \qquad (1.6)$$

The energy equation (1.1) for the nonconducting region ($r_d \le r \le r_w$) is brought by means of (1.2) and (1.3) to the form

$$s^\circ = s_w{}^\circ - EI \ln \eta / 2\pi s_0. \qquad (1.7)$$

Hence, when $\eta = \eta_d$, we obtain

$$\eta_d = \exp\left[-2\pi (s_d - s_w)/EI\right], \qquad (1.8)$$

where, according to Eqs. (1.5) and (1.6),

$$EI = \frac{2\pi (s_0 - s_d) F_1(\eta_d)}{F_2(\eta_d)}. \qquad (1.9)$$

Corresponding to s_d there is a particular temperature T_d at which the electrical conductivity σ is practically zero, so that the contribution to I from the region $T_d \ge T \ge T_w$ is negligibly small. For air we take T_d = 4000° K.

Assigning values of T_0, T_w, and p for the particular gas, we can use (1.4)-(1.9) to determine numerically, by the method of successive approximations, the distributions $s^\circ = s^\circ(\eta, p)$ and the values of Er_w, I/r_w, EI, and η_d. After this it is easy to find the enthalpy ($h^\circ = h/h_0$) or temperature ($T^\circ = T/T_0$) characteristic.

2. For an approximate determination of the velocity profile and other gasdynamic characteristics in the case of stabilized electric-arc heating we will assume that the ratio of the pressure to the velocity head on the axis is fairly large. Then, as will be explained later, we can ignore the second term on the right side of Eq. (1.1).

We write Eq. (1.1) in dimensionless form,

$$\frac{\partial v_z{}^\circ}{\partial \eta} = \alpha \frac{\eta}{\mu^\circ} + \frac{1}{\mu_0 v_{z0}} \frac{1}{\mu^\circ \eta} \frac{\partial \Lambda}{\partial z}, \qquad \alpha = \frac{r_w{}^2}{2\mu_0 v_{z0}} \frac{dp}{dz},$$

$$v_z{}^\circ = \frac{v_z}{v_{z0}}, \qquad \mu^\circ = \frac{\mu}{\mu_0}. \qquad (2.1)$$

After formal integration of Eq. (2.1) with respect to η with fulfilment of the boundary condition $v_z^\circ(0) = 1$ we obtain

$$v_z{}^\circ = 1 + \alpha F_3(\eta) + \beta F_5(\eta), \qquad \beta = \frac{r_w{}^2 \rho_0}{\mu_0} \frac{dv_{z0}}{dz},$$

$$F_3(\eta) = \int_0^\eta \frac{\eta d\eta}{\mu^\circ}, \qquad F_5(\eta) = \int_0^\eta \frac{E_4(\eta) \, d\eta}{\mu^\circ \eta},$$

$$F_4(\eta) = \int_0^\eta \rho^\circ (v_z{}^\circ)^2 \, \eta d\eta, \qquad \rho^\circ = \frac{\rho}{\rho_0}. \qquad (2.2)$$

We neglect the variation of the integrals F_3 and F_5 with z (this will be justified later). We assume that $\rho_0 v_{z0}$ = const. When $\eta = 1$ we have $v_z^\circ = 0$, and Eq. (2.2) gives

$$1 + \alpha F_3(1) + \beta F_5(1) = 0. \qquad (2.3)$$

Neglecting the temperature variation, we can regard the density as proportional to the pressure, since the molecular weight of the gas does not depend greatly on the pressure. Hence,

$$\rho_{01}/\rho_0 = v_{z0}/v_{z01} = 1/p^\circ, \qquad p^\circ = p/p_1. \qquad (2.4)$$

Using (2.4), we get Eq. (2.3) in the form

$$k_1 p^\circ dp^\circ - dp^\circ/p^\circ + 4k_2 z^\circ = 0,$$

$$k_1 = \frac{p_1 F_3(1)}{2\rho_{01} v_{z01}^2 F_5(1)}, \qquad k_2 = \frac{1}{RF_5(1)},$$

$$R = \frac{2r_w \rho_{01} v_{z01}}{\mu_{01}}, \qquad z^\circ = \frac{z}{2r_w}. \qquad (2.5)$$

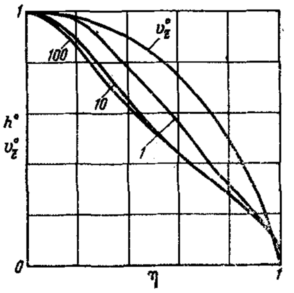
Table 1

	$10^{-5}p/1.01325$, N/m²	1	2	5	10	20	50	100
	s_0	2554	2275	2032	1893	1787	1674	1598
	σ_0	120.2	103.5	85.11	73.28	57.54	46.77	36.73
	s_d	0.3560	0.3800	0.3925	0.3930	0.3890	0.3827	0.3810
	$\overset{\circ}{s}_w$	0.0004	0.0105	0.0118	0.0127	0.0135	0.0143	0.0150
Initial Values	Er_w	20.2	21.2	22.8	24.1	25.6	27.9	30.0
	I/r_w	458	375	305	269	237	207	189
	$10^{-3}EI$	9.25	7.96	6.95	6.48	6.11	5.76	5.62
	η_d	0.548	0.518	0.500	0.498	0.502	0.511	0.519
Final Values	Er_w	15.9	17.6	20.3	22.9	25.7	29.9	33.5
	I/r_w	732	538	380	298	243	190	165
	$10^{-3}EI$	11.62	9.30	7.71	6.90	6.30	5.78	5.55
	η_d	0.618	0.567	0.533	0.518	0.510	0.511	0.517

The values of α and β, in view of (2.5), will be

$$\alpha = -\frac{k_1 (p^\circ)^2}{[k_1 (p^\circ)^2 - 1] F_3(1)}, \qquad \beta = \frac{1}{[k_1 (p^\circ)^2 - 1] F_5(1)}. \tag{2.6}$$

The values of $F_3(1)$ and $F_5(1)$ are of the same order. In particular, when $\rho^\circ = \mu^\circ = 1$ we will have $d\Lambda/dz = 0$ and then we will obtain a



Poiseuille flow, for which, as is known [4], $dp/dz = \text{const}$ and $v_z^\circ = 1 - \eta^2$.

In this case $F_3(1)/F_5(1) \approx 3.3$.

It is clear from (2.6) that at sufficiently large k_1 we can actually ignore the terms in Eqs. (1) and (2.1)–(2.3) which are due to the longitudinal velocity gradient.

We can estimate the variation of the pressure with z by integrating Eq. (2.5). We can either completely discard the second term, or replace $\ln p^\circ$ by $-(1 - p^\circ)$. In the latter case we obtain

$$p^\circ = [1 + \sqrt{1 + k_1 (k_1 - 2 - 8k_2 z^\circ)}]\, k_1^{-1}. \tag{2.7}$$

Hence, it is clear that at sufficiently large k_1 and relatively small z° the pressure along the tube will not decrease significantly. In consequence of (2.4) the axial velocity will also increase insignificantly. This, together with the previously mentioned smallness of the longitudinal variation of the pressure, justifies the assumption made in the derivation of Eq. (2.2) that the integrals F_3 and F_5 are independent of z.

On substituting the values of v_z from (2.2) in Eq. (1.3) we obtain

$$\gamma = F_6(1) + \alpha F_5(1) + \beta F_8(1),$$

$$\gamma = \frac{G}{2\pi \rho_0 v_{z0} r_w^2}, \quad F_6(1) = \int_0^1 \rho^\circ \eta \, d\eta, \quad F_7(1) = \int_0^1 \rho^\circ F_3(\eta)\, \eta \, d\eta,$$

$$F_8(1) = \int_0^1 \rho^\circ F_5(\eta)\, \eta \, d\eta. \tag{2.8}$$

Since on the basis of (2.2)

$$\tau_w = -\mu_w (\partial v_z/\partial r)_w = -\mu_0 v_{z0} r_w^{-1} [\alpha + \beta F_4(1)],$$

then the friction drag coefficient

$$\zeta = 8\tau_w/\rho_0 v_{z0}^2 = -8\mu_0 [\alpha + \beta F_4(1)]/r_w \rho_0 v_{z0}.$$

In view of (2.8), we obtain

$$\delta = -\gamma\, [\alpha + \beta F_4(1)] \qquad (\delta = G\zeta/16\pi r_w \mu_{01}). \tag{2.9}$$

The start of stabilized electric-arc heating, i.e., the case $p^\circ = 1$, is of greatest interest. In such a case, on the basis of (2.6)

$$\alpha = -k_1/(k_1 - 1)F_3(1), \qquad \beta = 1/(k_1 - 1)F_5(1). \tag{2.10}$$

At large k_1 the values of α, γ, δ will be given by the formulas

$$\alpha = -\frac{1}{F_3(1)}, \quad \gamma = \frac{F_3(1) F_6(1) - F_7(1)}{F_3(1)}, \quad \delta = -\alpha\gamma. \tag{2.11}$$

A common characteristic of electric-arc heating is the mean bulk enthalpy. Proceeding from the expression

$$\langle h \rangle = \int_0^{r_w} \rho v_z h r \, dr \left(\int_0^{r_w} \rho v_z r\, dr \right)^{-1}$$

we arrive at the following formula for $\langle h^\circ \rangle = \langle h \rangle / h_0$:

$$\langle h^\circ \rangle = \frac{1}{\gamma} \int_0^1 \rho^\circ v_z^\circ h^\circ \eta \, d\eta. \tag{2.12}$$

3. In application to a particular case we consider the assumption

$$\rho v_z = \text{const}. \tag{3.1}$$

It follows from Eq. (1.3) that $\gamma = 1/2$. In view of (3.1) and the fact that the value of ρ_w is bounded, the condition $v_z^\circ(1) = 0$ will not satisfied; instead, according to (3.1), we obtain

$$v_z^\circ(1) = 1/\rho_w^\circ.$$

Hence, the right side of the equation, similar to (2.3), will be equal to $1/\rho_w^\circ$.

At large k_1 the terms with β can be neglected and then

$$\alpha = -(\rho_w^\circ - 1)/\rho_w^\circ F_3(1), \qquad \delta = -\tfrac{1}{2}\alpha.$$

In this case the mean bulk enthalpy can be expressed on the basis of (2.12),

$$\langle h^\circ \rangle = 2 \int_0^1 h^\circ \eta \, d\eta. \tag{3.2}$$

4. We carried out calculations for air with an axial temperature of 6000° K and pressures 1–100 atmospheres. We used known data for the temperature and pressure dependences of the thermal conductivity and viscosity [5], the electrical conductivity [6], and the enthalpy and density [7].

As an initial approximation for σ we took the approximating expression $\sigma^\circ = 1 - 6(\eta/\eta_d)^2 + 8(\eta/\eta_d)^3 - 3(\eta/\eta_d)^4$, which satisfied the conditions

$$\sigma^\circ(0) = 1, \quad \sigma^\circ(\eta_d) = 0,$$

$$(d\sigma^\circ/d\eta)_{\eta=0} = (d\sigma^\circ/d\eta)_{\eta=\eta_d} = (d^2\sigma^\circ/d\eta^2)_{\eta=\eta_d} = 0.$$

This approximation enable us to determine the functions $F_1(\eta)$ and $F_2(\eta)$ in final form, to calculate the characteristics Er_w, I/r_w,

Table 2

$10^{-5}\,p/1.01325$, N/m²		1	2	5	10	20	50	100
First approximation ($\beta = 0$)	$-\alpha$	1.202	1.160	1.116	1.086	1.066	1.055	1.050
	γ	0.537	0.559	0.594	0.609	0.618	0.626	0.629
	δ	0.647	0.655	0.659	0.660	0.660	0.660	0.660
Second approximation	$-\alpha$	1.320	1.184	1.120	1.086	1.066	1.055	1.050
	β	0.271	0.059	0.009	0.002	0.000	0.000	0.000
	γ	0.554	0.575	0.596	0.609	0.618	0.626	0.629
	δ	0.680	0.665	0.664	0.661	0.660	0.660	0.660
	$\langle h^\circ \rangle$	0.475	0.446	0.420	0.413	0.410	0.409	0.408
$\rho v_z = \text{const}$, $\beta = 0$	$-\alpha$	1.085	1.042	0.998	0.971	0.953	0.941	0.935
	δ	0.542	0.521	0.499	0.486	0.476	0.470	0.468
	$\langle h^\circ \rangle$	0.466	0.425	0.411	0.399	0.395	0.394	0.394

EI, η_d, and s° in an initial approximation from formulas (1.5), (1.6), (1.9), (1.8), and (1.4) and to determine $\sigma^\circ = \sigma^\circ(s^\circ)$ in a first approximation. We then carried out a numerical calculation of these functions and characteristics in a first approximation. In a similar way we found higher-order approximations. The distribution $s^\circ = s^\circ(\eta, p)$ for the nonconducting region was calculated from formula (1.7). Table 1 gives some of the data required for the calculations.

Calculations of the indicated characteristics show that it is sufficient to take the second approximation. In the calculations, however, approximations to the fourth order were made. The initial and final values of Er_W, I/r_W, EI, and η_d are shown in Table 1. Figure 1 shows the enthalpy profiles $h^\circ = h^\circ(\eta, p)$ found from the distributions $s^\circ = s^\circ(\eta, p)$.

We obtained corresponding distributions $\rho^\circ = \rho^\circ(\eta, p)$, $\mu^\circ = \mu^\circ(\eta, p)$ which enabled us to determined numerically the functions $F_3(\eta, p)$, $F_6(\eta, p)$, and $F_7(\eta, p)$, and to calculate the gasdynamic characteristics α, γ, and δ in a first approximation ($\beta = 0$) from formulas (2.11). We also determined the velocity profiles $v_z^\circ = v_z^\circ(\eta, p)$ from formula (2.2) ($\beta = 0$). The values of α, γ, and δ are given in Table 2. We found that the velocity profile is independent of the pressure to an accuracy of two to three decimal places. Figure 1 shows v_z° as a function of η.

The obtained values of γ show that at constant flow rate G and radius r_W the product $\rho_0 v_{z0}$ decreases slightly with pressure increase. Since the density is almost proportional to the pressure, the velocity v_{z0} will decrease rapidly, and the coefficient k_1 will increase rapidly, with pressure increase. Thus when G = const and r_W = const, β will presumably approach zero rapidly with pressure increase; this is confirmed by calculation (Table 2).

We next calculated the functions $F_4(\eta, p)$ and $F_5(\eta, p)$ and the parameters α, β, γ, and δ in a second approximation from formulas (2.10), (2.8), and (2.9). The results are also given in Table 2.

Although when G = const and r_W = const the value of $\rho_0 v_{z0}$ decreases slightly with pressure increase (in the considered pressure range the decrease does not exceed 12%), we assumed in the calculations of the coefficients k_1 that $\rho_0 v_{z0}$ = const = 21.59. This value is obtained when $T_0 = 6000°$ K, $p = 1.01325 \cdot 10^5$ N/m², v_{z0} = = 482 m/sec (which corresponds to $M_0 = 0.3$, at which the kinetic energy can still be neglected in comparison with the enthalpy).

A comparison of the results of calculation in the first and second approximations shows that for the considered conditions the effect of the variation of v_{z0} with z is insignificant and is practically absent at pressures $p \geq 2 \cdot 1.01325 \cdot 10^5$ N/m². Since for the first two pressures in Table 2 the values of α in the second approximation are larger, the corresponding velocity profiles $v_z^\circ = v_z^\circ(\eta, p)$ (with β taken into account) are practically the same as the profiles obtained in the first approximation ($\beta = 0$). For comparison Table 2 gives the values of α and β ($\beta = 0$) on the assumption that ρv_z= const. As we observed, in this case $\gamma = 0.5$, irrespective of the pressure. These values of α, γ, δ differ from the values in the first approximation by 9.7–10.9%, 6.9–20.5%, and 15.2–29.0%, respectively, and the difference increases with pressure increase. Table 2 also gives the values of the dimensionless mass mean enthalpy $\langle h^\circ \rangle$ calculated from formulas (2.12) and (3.2). The difference between corresponding values is 1.9–4.5%.

In conclusion we evaluate the changes in pressure for r_W = 0.001 m, $p_1 = 1 \cdot 01325 \cdot 10^5$ N/m², and $T_0 = 6000°$ K. In this case k_1 = 11.2 and k_2 = 0.00985. Calculation from formula (2.7) with $z^\circ = 5$ gives $p^\circ = 0.985$.

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